A sufficient condition for spiral cone beam long object imaging via backprojection

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Abstract

The response of a point object in cone beam spiral scan is analysed. Based on the result a sufficient condition for the spiral scan long object problem employing backprojection is formulated. By making use of the sufficient condition a general class of exact, backprojection based reconstruction algorithms for spiral scan cone beam CT is developed which are capable of reconstructing a sectional ROI of the long object without contamination from overlaying materials using spiral scan cone beam data irradiating the particular ROI and its immediate vicinity only. Also, at each source position the minimum size of the region on the detector plane required for 3D backprojection is reduced, which in term brings about reduction in the amount of 3D backprojection computation.

I. 2D filtering and masking

Spiral scan computed tomography with large area detectors is of increasing interest for rapidly scanning spacious volumes. As the cone angle increases the artifacts generated in the reconstructed images by the approximate reconstruction algorithms will become more and more serious, and exact reconstruction algorithms are required. It is known that if the spiral path is long enough so that every plane intersecting the object also intersects the spiral path, the object can be reconstructed. For long objects, however, it is highly desirable to scan only the portion of the object that is of interest, for the sake of reduction in scan time as well as radiation protection of the patient in medical imaging. However, as a consequence of the divergent nature of the X-ray cone-beams different regions of the object are correlated. To reconstruct only a region-of-interest (ROI) from a spiral scan which covers the particular ROI and its immediate vicinity only poses a challenge for the imaging community. This is referred to as the long object problem in the literature.

The first solution to the long object problem in spiral cone beam CT is the Radon space driven (spiral + 2 circles) algorithm reported in [1,2]. A key part of the reconstruction algorithm is the data-combination technique in which the radial Radon derivative for each plane intersecting the ROI is obtained by combining the partial results computed from the cone beam data at the various source positions that the plane intersects. The method is illustrated in Figure 1 which represents a plane Q intersecting the ROI and the scan path. Since the partial planes do not overlap and together they completely cover the portion of plane Q that lies within the ROI, the Radon derivative for plane Q can be obtained exactly by summing the Radon derivatives for the partial planes. From Figure 1 it is evident that the portions of the object outside the ROI do not need to be irradiated. Therefore during scanning collimators can be used to block off radiation from reaching those portions.

Restricting the cone beam projection data to the appropriate angular range for data combination can be accomplished by a masking process. The mask consists of a top curve and a bottom curve formed by projecting on the detector the spiral turn above and the turn below from the current source position. It can be easily seen that such masking procedure corresponds exactly to the angular range bound by the prior and the subsequent source positions as indicated in Figure 1. We shall refer to this mask as the data-combination mask. For a flat detector located at the rotation axis such that the line connecting the source to the detector origin is normal to the

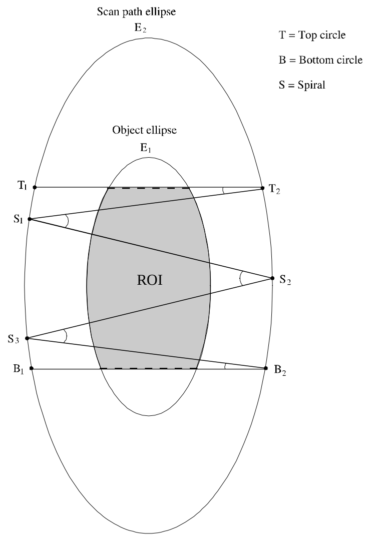

Figure 1. A typical integration plane covering the ROI defined by the source positions. Other integration planes may have more or less spiral scan path intersections, and may not intersect either the top or the bottom circle scan paths.

detector plane, the equation for the top curve for the spiral scan is given by:

$$v = \frac{h}{2\pi} \tan^{-1}\left(\frac{R}{u}\right)\left(1 + \frac{u^2}{R^2}\right) \qquad u \geq 0$$

$$v = \frac{h}{2\pi}\left[\pi + \tan^{-1}\left(\frac{R}{u}\right)\right]\left(1 + \frac{u^2}{R^2}\right) \qquad u < 0 \tag{1}$$

where u and v are the Cartesian coordinate axes of the detector with the v axis coinciding with the rotation axis, R is the radius of the spiral, and h is the distance between adjacent spiral turns (the pitch). The bottom curve is the reflection of the top curve about the origin, i.e. $(u,v) \rightarrow (-u,-v)$. The shape of the spiral mask is shown in Figure 2. The figure assumes right-handed spiral rotation.

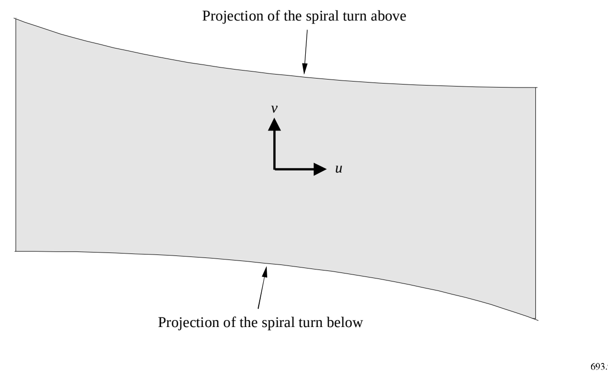

Figure 2. The mask on the flat detector which defines the desired partial plane for Radon derivative computation. For any plane of integration, the portion of its intersection line with the detector within the mask is the desired partial plane.

In the backprojection version of the (spiral + 2 circles) algorithm [3], the masked cone beam data are 2D filtered and then 3D backprojected. The 2D filtering is carried out in 2 different manners for different parts of the cone beam data: the data inside the mask are line-by-line ramp filtered in the direction of the projected scan path direction, whereas those on the mask boundary are processed with a 2D filter which includes 2D backprojection at all angles on the detector plane. By virtue of the Radon inversion formula the 2D backprojection operation should be extended to the entire detector plane extended to infinity; in practice it is extended to the extent sufficient to cover the entire ROI. Through the line-by-line ramp filtering in the direction of the projected scan path direction the data inside the mask boundary only affect a localized portion of the reconstruction volume. On the other hand the data on the mask boundary affect the entire ROI because of the long range of the 2D backprojection. This long range correlation caused by the mask boundary data is the crux of the long object problem employing backprojection driven algorithms.

II. Spiral scan long object problem

Recently a number of approaches solving the long object problem with only the spiral scan appeared in the literature. In the virtual circle (VC) method reported by Kudo et al [4] it is found that by utilizing the unique property of the PI lines [5], removing the circles in the (spiral + 2 circles) algorithm contaminates only a localized portion at each end of the ROI, and thus the remaining portion of the ROI can still be reconstructed without contamination from overlaying materials. In the zero boundary (ZB) method reported by Defrise et al [6], the unique property of the PI lines is also utilized to remove the long range correlation between different regions of the object caused by the data on the mask boundary. In the local ROI (LR) method developed by Sauer et al [7] and later implemented by Schaller et al [8], de-correlation between different regions of the object is achieved on the ϕ-planes, which are the planes which contain the *z* axis in the Radon space. Subsequently the backprojection version of the local ROI method was developed by Tam [9] and implemented by Lauritsch et al [10]. Unlike the (spiral + 2 circles) algorithm, with only spiral scan it is necessary to scan some portions of the object adjacent to the ROI in order to reconstruct the ROI without contamination; the spiral path required beyond the ROI is referred to as overscan in the literature. For comparison the overscan for the (spiral + 2 circles) algorithm is zero.

All approaches are theoretically exact solutions to the long object problem. However, even among the backprojection driven algorithms very different methodologies are employed in reconstructing the ROI without contamination, and the overscan required by each algorithm is substantially different from the others. It is not apparent that the three methods have any features in common.

III. A sufficient condition

In this paper a sufficient condition for backprojection driven image reconstruction algorithms for the long object problem with spiral scan is derived. The analysis is based on the analysis of the response of a point object enclosed inside the spiral path. It is found that the support of the contribution to the reconstruction volume from the cone beam data on the mask boundary becomes localized when certain condition is satisfied. Each mask boundary data point corresponds to a PI line, as illustrated in Figure 3, which intersects two source positions. At each of the 2 source positions the mask boundary data point, after some processing, is 2D backprojected along each line intersecting the data point, and then 3D backprojected onto the 3D backprojection planes defined by the source position and each 2D backprojection line through the data point. Thus each 3D backprojection plane intersects the line connecting the source position and the data point, which is the PI line corresponding to the data point. Since the 2 source positions that acquire the mask boundary data point have the same PI line, it follows that the 2 source positions have the same set of 3D backprojection planes when processing the data point.

Consider a fixed mask boundary data point. If for each 3D backprojection plane the data point is processed, which includes filtering and 3D backprojection, to the same extent at the 2 source positions that acquire the data point, then the support of the contribution to the reconstruction volume from the data point can be shown to be localized. The minimum size of the region on the detector plane required for 2D backprojection and the subsequent 3D backprojection for these mask boundary data can be prescribed using projective geometry, and is found to be smaller than the minimum size according to current understanding, viz. the size required to cover the entire ROI. The extent to which the detector size is reduced depends on the spiral pitch, and is substantial for small pitch. The reduction in the detector size is important for the reduction in the amount of 3D backprojection computation.

Among the three above-mentioned long object backprojection driven reconstruction algorithms, the VC method and the backprojection LR method are found to satisfy the sufficient condition, but not the ZB method. Based on the sufficient condition a general class of exact, backprojection driven reconstruction algorithms for long object imaging in spiral scan cone beam CT is developed. It is found that the VC method is a special case of this class of algorithms.

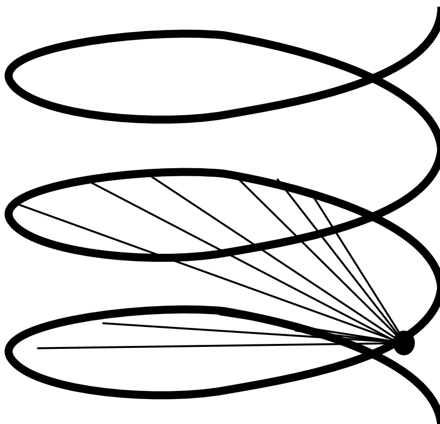

Figure 3 Mask boundary data and the corresponding 2 source positions on the PI lines.

IV . References

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